ADVANCED CONNECTIONS | CYSYLLTIADAU PELLACH

PHYSICS MENTORING PROJECT | PROSIECT MENTORA FFISEG

PRE-TUTORIAL MATERIAL

SESSION 5: FUNCTIONING LIKE A PHYSICIST

In this session we'll look at exponentials and logarithms, how they relate to each other, and why they're relevant in physics.

EXPONENTIAL TRENDS

Lets say that some value, A, changed over time. We would write this rate of change as:

$$\frac{dA}{dt}$$

The variable t doesn't necessarily need to be time (but it often is)

EXAMPLE: If A were **velocity,** its rate of change (the expression above) would simply be **acceleration -** and we know lots of equations to describe an object with constant acceleration!

But what if A changed by a constant **factor** over time, instead of a constant amount?

EXAMPLE: If the factor was 2, A would **double** every unit of time

$$\frac{dA}{dt} = kA$$

For a general case, we use **k** to represent the factor

Using a bit of calculus, we can put this in terms of A:

$$A = A_0 e^{kt}$$

The method to find this is in the optional extras!

$A =$ amount
$A_0 =$ initial amount
$k =$ decay constant
$t =$ time interval

This is the equation for **exponential growth/decay:**

GROWTH happens if **k>0**

DECAY happens if **k<0**

TASK 1: FIND THE ODD ONE OUT

5 MINS

One of the data sets in the following table (A, B, or C) is **not** changing exponentially with time. **Which one is it?**

t (s)	0	15	30	45	60	75	90
A	20	22	24	26	28	30	32
B	0.01	0.1	1	10	100	1,000	10,000
C	100	90	81	73	66	59	53

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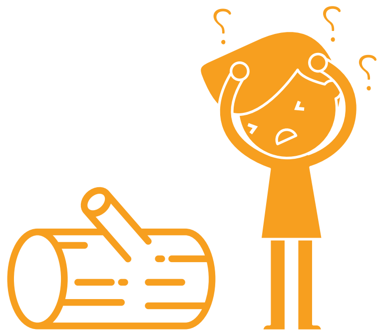
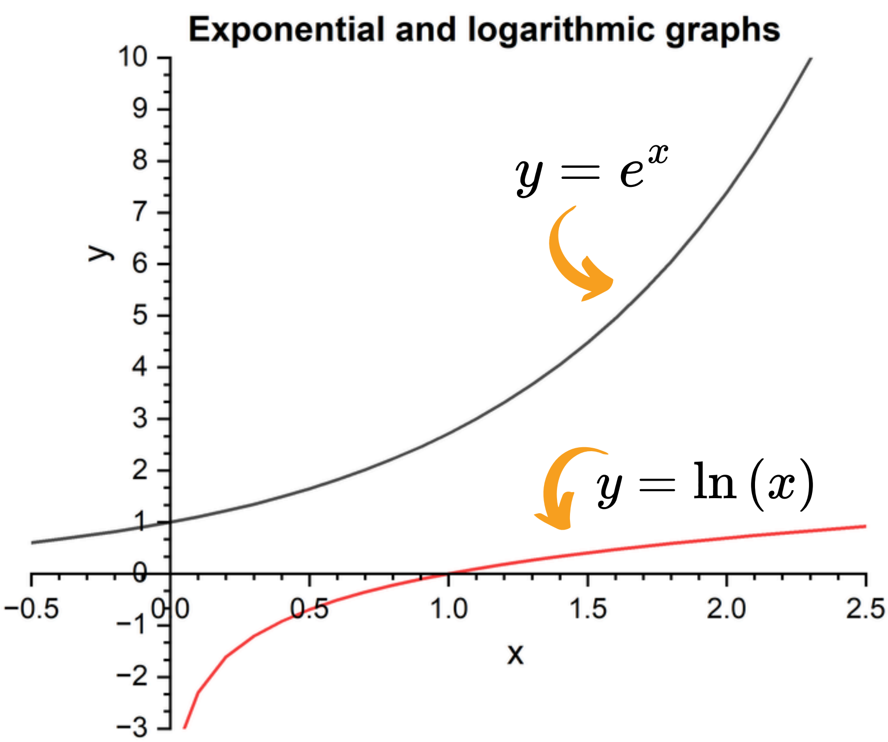
LOGARITHMS

Logarithms allows us to alter an exponential expression so it can be written in terms of the **power (kt).** This is often the value of interest. The equations below show how we would go **from an exponential expression to a logarithm.** Aloud, this would be said as 'log base A of y is x'.



$$y = A^x \quad => \quad log_A(y) = x$$

What do these functions look like on a graph?



What's the deal with e and ln?

The constant **e** comes about during the calculus step mentioned before, and it **comes up a lot** in sciences - especially **physics,** which relies heavily on maths. A **logarithm with base e** is called a **natural logarithm** and is expressed as **ln.**

Some useful log rules

$$log(a) + log(b) = log(ab)$$

$$log(a) - log(b) = log\left(\frac{a}{b}\right)$$

TASK 2: LOG PROBLEMS

10 MINS

Find x in the following equations:

1. $log_{10}(x) + log_{10}(50) = 3$
2. $log_x(64) = 6$
3. $e^x = 20$

HINT: use the log rules to help you!

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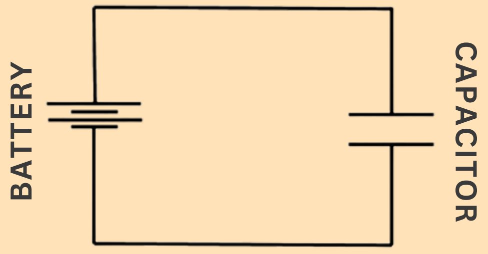
WHERE DOES THIS COME INTO PHYSICS?

This sort of mathematics comes up in science all the time! For A-level physics, there are two topics where logarithms/exponentials come up: **capacitance** and **radioactive decay**.

CAPACITANCE

A **capacitor** is a component of an electronic circuit consisting of **two metal plates** separated by an **insulator.** Each plate is connected to one end of a battery or cell, causing **electrons to flow** from one plate to another.



As one plate becomes more **positively** charged, it becomes **harder to remove the negative electrons** from it, just as it becomes harder to **add** them to the negatively charged plate. The **current** therefore depends on the **charge** of the capacitor in an **exponential** relationship:

$$Q = Q_0 e^{\frac{-t}{RC}}$$

$Q =$ charge remaining
$Q_0 =$ initial amount of charge
$R =$ resistance
$C =$ capacitance
$t =$ time interval

RADIOACTIVE DECAY

The nucleus of an atom is made of **protons** (positive charges) and **neutrons** (neutral charges). We know from electrostatics that these should repel, but there is another force - the **strong nuclear force** - which holds nuclei together. This force can only act over short distances though, so **large nuclei** are much more **unstable.**

Radioactive decay is when an unstable nucleus **emits particles** (nuclear radiation) to become more stable. We don't know when a nucleus will decay, only the **probability** that it will over a certain period of time. This means that the **number of atoms that decay per second** is proportional to the **total number of remaining atoms** in an **exponential** relationship:

$$N = N_0 e^{-\lambda t}$$

$N =$ no. of atoms
$N_0 =$ initial no. of atoms
$\lambda =$ decay constant
$t =$ time interval

HINT
$\ln\left(e^x\right) = x$

TASK 3: REAL PROBLEMS

15 MINS

How would you express the two equations above in terms of t?

OPTIONAL EXTRAS

HOW DID WE GET THE EQUATION ON THE FIRST PAGE?

$$\frac{dA}{dt} = kA$$

$$\int \frac{1}{A} dA = k \int dt$$

We can look this up in integration tables

$$\ln(A) = kt + C$$

$$A = e^{kt+C} = e^C \times e^{kt} = A_0 e^{kt}$$

We simply define this constant as the value of **A** when **t=0**

LOUDNESS - DECIBELS

We've all heard loudness measured in **decibels (dB)**, but did you know that decibels actually use a **logarithmic scale?** This means that 2 decibels louder is **twice as loud!**

$$\beta = 10 log_{10}\left(\frac{I}{I_0}\right)$$

The reference intensity is the quietest sound a human can hear:

$$I_0 = 10^{-12} Wm^{-2}$$

$\beta =$ loudness (dB)
$I =$ intensity of the sound wave
$I_0 =$ reference intensity

Normal conversation occurs at about **60 dB,** whereas a jet taking off 30 m away is **140 dB.** What is the **difference in intensity** of sound waves between these two sounds?

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OPTIONAL
EXAM PRACTICE

All exam questions in Advanced Connections are taken from WJEC A-level Physics papers!

Have a go at some of these exam questions:

1. *(a)* Radon gas decays by emitting α-particles. It has a half-life of 3.8 days. Calculate the **percentage reduction** in the activity of a sample of radon after 12 days. [4]

(b) A student makes the following measurements for a radioactive source using the indicated absorber between the source and detector.

Absorber	Counts per minute
none	1 004
sheet of paper	597
2 mm of aluminium	23
15 cm of lead	27

Explain these observations. [4]

2. *(a)* A radioactive sample of material has a half-life of 11.4 days and an initial activity of A_0. Determine:

(i) the decay constant; [2]

(ii) the activity of the sample after 57.0 days in terms of A_0; [2]

(iii) the **percentage decrease** in the number of nuclei in the sample after 57.0 days. [3]

TOP TIP

Exam questions are the most effective way to revise for your exams (who would have guessed...!). By practising your exam technique, you'll soon see there are **patterns** to the questions and how they want you to respond. You might even notice **similar questions** popping up again and again!

You can speak to your teacher and ask them to **mark a past paper** for you if you want to have a practice run.