

## PRE-TUTORIAL MATERIAL

### SESSION 3: ANALYSING LIKE A PHYSICIST

Last session we considered **accuracy** and **precision**. These are very important concepts in physics - and very important for this session, in which we'll be looking at **errors** and **uncertainty**.



## WHY IS UNCERTAINTY IMPORTANT?

Every measurement or reading has some **uncertainty** associated with it. In most cases, this can be reduced - but it can never be eliminated entirely. Keeping track of these uncertainties is very important, because we cannot compare results without them.

## EXAMPLE: RADIUS OF A PROTON

### PERCENTAGE UNCERTAINTY

Until 2010, the **radius of a proton** was agreed upon as being **0.877 fm**. This value came from **two separate methods of measurement**:



**REMEMBER:**

1 fm = 1/1,000,000 of a nm

#### 1. THE SPECTROSCOPY METHOD

0.8768 fm  
± 0.8%

0.8775 fm  
± 0.06%

#### 2. THE ELECTRON-PROTON SCATTERING METHOD

Clearly these two values are **different**, but if we take into account the range of values they each represent according to their **percentage uncertainty**, we find that they actually agree!

In 2010, a **new method** with even greater precision was used to find the proton radius. This gave a value of **0.842 fm ± 0.1%**. This does not agree with the previous values, but **we only know this because the uncertainties are low enough** to show it. **This is an unsolved problem in physics!**

## ABSOLUTE UNCERTAINTY

Although percentage uncertainties are useful for **comparing the relative uncertainties** of different values, It isn't immediately clear whether the values above actually agree or not. To confirm this we will need to look at **absolute uncertainty** instead.

### WHAT IS 0.8% OF 0.8768?

$$0.8768 \times \frac{0.8}{100} = 0.007 \text{ fm}$$

So we can write:

$$0.8768 \pm 0.007 \text{ fm}$$

In other words, the value could lie anywhere between **0.8779 and 0.8793 fm**. The value from the second method already lies in this range so **we don't need to check** its absolute uncertainty to know that they agree!

## TASK 1: PROVE IT!

By finding the **absolute uncertainties** of the other two values, show **10 MINS** that the value found in 2010 **does not agree** with the other two.

### PROPAGATION OF UNCERTAINTY

Why do we use percentage uncertainty?

- We can **compare the precision** of values
- This type of uncertainty is very useful in **uncertainty propagation**



This is where we **combine uncertainties**. For example, if we know the uncertainty in a measured value  $x$  in an experiment, we can **propagate** to find the uncertainty of  $y$  where  $y = mx + c$

**EXAMPLE 1** This is Hooke's Law!

$F = kx$  where  $k$  is a constant. Because  $k$  is a constant, it **doesn't have an uncertainty**. This means that the **percentage uncertainties of  $F$  and  $x$  are the same!**

**EXAMPLE 2**

when calculating in this way with percentage uncertainties, we **ignore constants**

If there are **multiple uncertainties**, then we need to propagate them by **adding them together, regardless of whether we're multiplying or dividing**

$P = IV$

This is Watt's Law!

$P = ?$

$I = 2 \text{ A} \pm 1\%$

$V = 35 \text{ V} \pm 5\%$

$P = (2 \times 35) \pm (1\% + 5\%)$

$P = 70 \pm 6\% \text{ W}$

If the equation involves **addition** or **subtraction**, we do exactly the same as Example 2, except we use the **absolute** uncertainty instead of the percentage uncertainty.

### TASK 2: PROPAGATION PRACTICE

15 MINS

#### 1. POWER EQUATION

Calculate the values and their uncertainties!

$$P = \frac{W}{t}$$

$P = ?$

$W = 5 \text{ kJ} \pm 5\%$

$t = 5 \text{ mins} \pm 4\%$

#### 2. EXTENSION OF A SPRING

$$F = kx$$

$F = ?$

$k = 60 \text{ N/m}$

$x = 75 \text{ cm} \pm 2\%$

#### 3. MAXIMUM KINETIC ENERGY OF AN EMITTED PHOTOELECTRON

$$E_k = hf - \phi$$

$E_k = ?$

$hf = 9.8 \text{ eV} \pm 0.8\%$

$\phi = 5.1 \text{ eV} \pm 2\%$

## SIGNIFICANT FIGURES

The **number of significant figures** (or “sig figs”) is important when using uncertainties.

### EXAMPLE

If we measure a distance of **591.5 m ± 10 m**, then every number after the 9 in that value is **useless** because they are very small compared to the error!

We want to make sure uncertainty is the same **precision** as the measured value: **590 ± 10 m**

If a value doesn't have an uncertainty written, we **assume** it's ± the final sig fig. For example, 3.2 V would have an **assumed absolute uncertainty** of ± 0.1

## TASK 3: SIG FIG PRECISION

5 MINS

1 What would be the **assumed absolute uncertainty** of the value **A = 1093.6?**

2 How would you write the **speed of light** with its **absolute uncertainty** if its percentage uncertainty was ± 0.1%?

**HINT:**  
 $c = 299792458 \text{ m/s}$

## REDUCING UNCERTAINTY

It is very important to **reduce uncertainty** in experiments wherever we can.

### REDUCING ABSOLUTE UNCERTAINTY

More precise measuring techniques

### REDUCING PERCENTAGE UNCERTAINTY

Making the precision of the equipment small compared to the measured values (e.g. dropping a ball from a larger height)

We can reduce all uncertainties by repeating experiments!

## TASK 4: EXPERIMENTS

5 MINS

1 Consider an experiment in which we're trying to measure acceleration due to gravity by dropping a ball a distance of 1 m and using a stopwatch to measure the time it takes to hit the ground. The ruler has an uncertainty of ± 1 mm and the stopwatch ± 0.01 s.

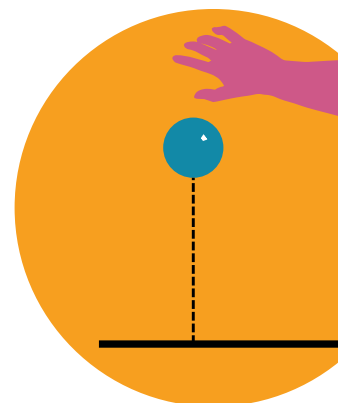
**What are the greatest sources of uncertainty, and how would you alter the experiment to make it more precise?**

2 When we repeat an experiment, we can use the **spread of results to find our uncertainty**.

You need to find the diameter of a cylinder that **isn't perfectly straight**. Even if you use a super-precise measuring device, you might get a value that is too large or too small depending on where you take your measurement. You take 5 measurements at different points:

2.577 m   2.581 m   2.585 m   2.583 m   2.574 m

**What is the average diameter?** Can you calculate the **absolute and percentage uncertainties** using the range of the results?



## OPTIONAL EXTRAS

### DETECTING GRAVITATIONAL WAVES

A breakthrough in 2015 saw us detect gravitational waves for first time **ever** with the LIGO (Laser Interferometer Gravitational-Wave Observatory).

#### WHAT IS A GRAVITATIONAL WAVE?

Gravitational waves are **fluctuations in the fabric of spacetime** caused by interactions between massive celestial objects.

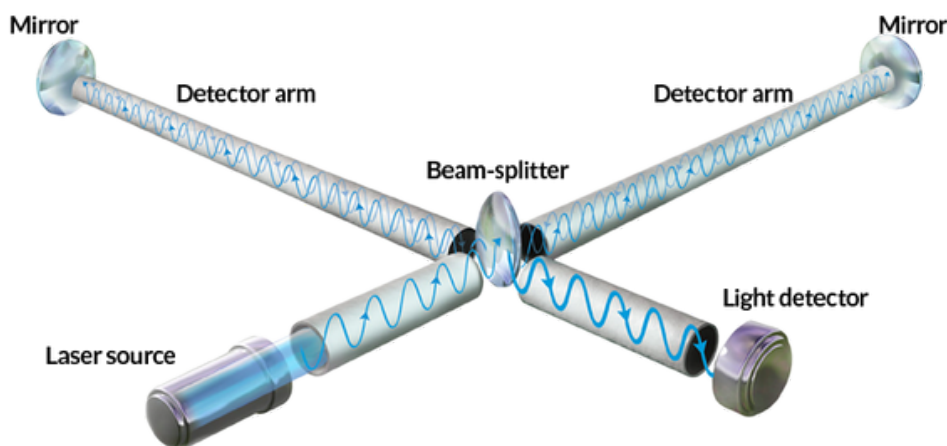
Black holes are usually the only things big enough to cause detectable waves!

The first detection was of two black holes **merging**. As the waves passed by the Earth, they caused a **minute change in distances**. The LIGO has to be able to detect a change **1,000 times smaller than a proton** over a distance of **4 km**?! The uncertainty had to be less than the change in distance in order for scientists to prove the existence of the gravitational wave.

What is the maximum percentage uncertainty that LIGO needs to be able to measure distance with?

**HINT:**  
We talked about the size of a proton earlier!

$$0.8768 \pm 0.007 \text{ fm}$$



### HARDER UNCERTAINTY CALCULATIONS

If an equation involves **powers**, we use the following method:

$$s = ?$$

$$u = 5.5 \pm 0.1 \text{ m/s}$$

$$t = 45 \pm 1 \text{ s}$$

$$a = 10.8 \pm 0.2 \text{ m/s}^2$$

**1** Find the value of  $s$  and its absolute uncertainty:

$$s = ut + \frac{1}{2}at^2$$

$$F = kx^2$$

The **percentage uncertainty** of  $F$  is **2 x the percentage uncertainty of  $x$**

**2** Write an expression for the volume,  $V$ , of water in this swimming pool in terms of its length,  $L$ , its width,  $W$ , and its depth,  $D$ . Then find  $V$  and its error given that:

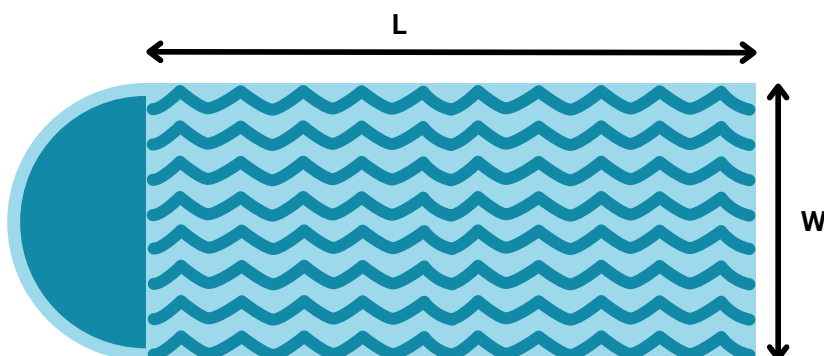
Why do you think the uncertainty for  $D$  is larger?

$$V = ?$$

$$L = 15 \pm 0.01 \text{ m}$$

$$W = 5 \pm 0.05 \text{ m}$$

$$D = 1.5 \pm 0.2 \text{ m}$$



### OPTIONAL EXAM PRACTICE

All exam questions in Advanced Connections are taken from WJEC A-level Physics papers!

Have a go at some of these exam questions:

1.

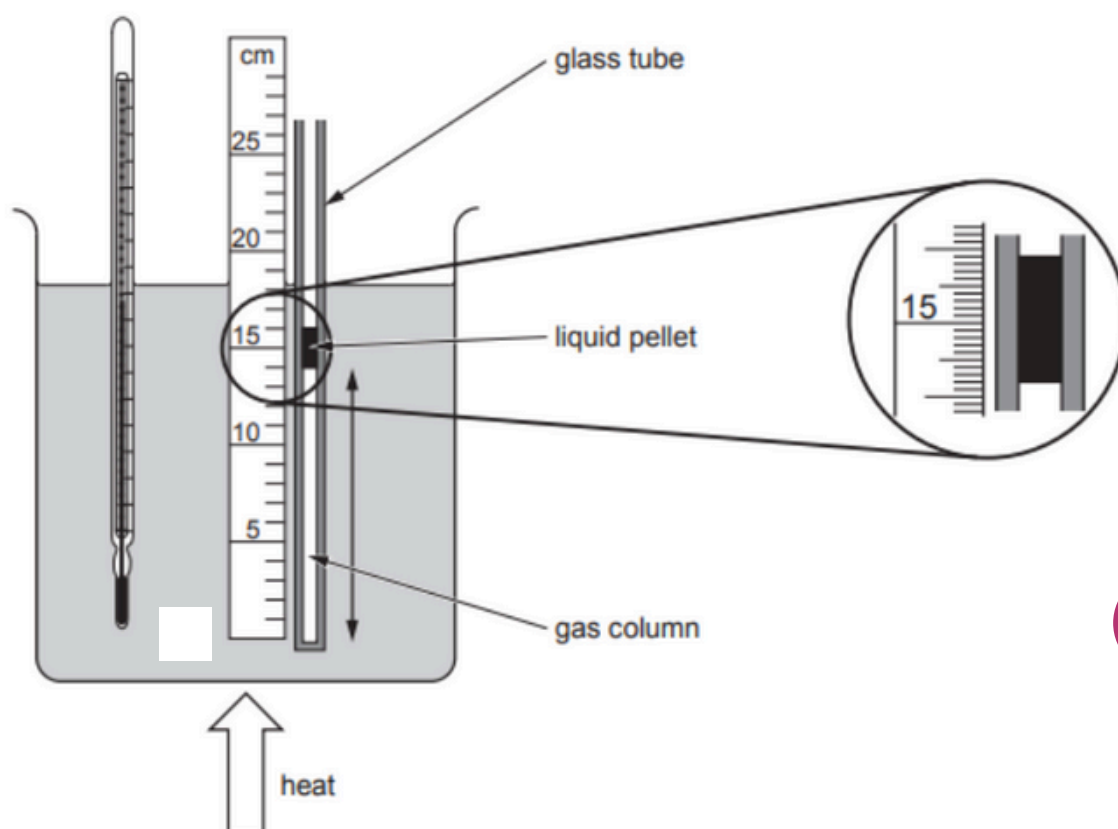
A student is given wire cutters and a reel of metal wire, and asked to find the resistivity of the metal. She cuts off a piece of the wire and makes measurements on it. She takes repeat readings and obtains the same values each time.

Quantity	Measurement	Instrument	Resolution of instrument
length	812mm	metre ruler	1 mm
diameter	0.48mm	digital calipers	0.01 mm
resistance	2.2 $\Omega$	digital multimeter	0.1 $\Omega$

- (a) (i) Calculate the resistivity of the metal of the wire. [3]
- (ii) Calculate the **absolute** uncertainty in the resistivity giving your value to an appropriate number of significant figures. [3]
- (b) Suggest **one** way in which the student could reduce the uncertainty in her value for the resistivity, using the same reel of wire and the same instruments as before. Explain briefly why the uncertainty would be reduced. [2]

2.

A student performs an experiment to estimate the Celsius temperature of absolute zero.





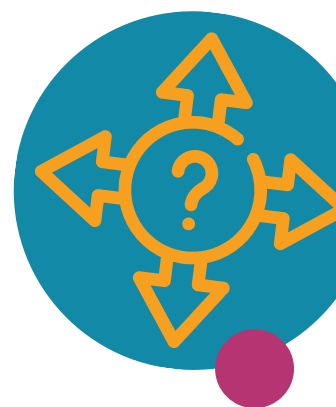
### OPTIONAL EXAM PRACTICE CONT.

Have a go at some of these exam questions:

2. The length,  $l$ , of the gas column and its temperature,  $\theta$ , are measured at atmospheric pressure,  $1.01 \times 10^5 \text{ Pa}$ . The temperature of the water is initially  $0.0^\circ\text{C}$ . It is then increased to  $80.0^\circ\text{C}$ . Readings are taken at  $20.0^\circ\text{C}$  intervals. The scale used to measure the length gives an uncertainty of  $\pm 0.1 \text{ cm}$  at the top of the column and an uncertainty of  $\pm 0.1 \text{ cm}$  at the bottom.

Values of  $\theta$  and  $l$  are recorded in the table below.

$\theta / ^\circ\text{C}$	$l / \text{cm}$
0.0	11.5
20.0	12.5
40.0	13.2
60.0	14.2
80.0	15.0



- (a) Justify the number of significant figures used to record the length,  $l$ . [2]
- (b) The glass tube in which the gas is trapped has an internal diameter of  $(1.5 \pm 0.1) \text{ mm}$ .
- (i) Calculate the volume of the gas at  $0.0^\circ\text{C}$ . [2]
- (ii) Show that the percentage uncertainty in this volume,  $V$ , is approximately 15%. [3]

reminder of the diagram!

