ADVANCED CONNECTIONS | CYSYLLTIADAU PELLACH

PHYSICS MENTORING PROJECT | PROSIECT MENTORA FFISEG

PRE-TUTORIAL MATERIAL

SESSION 5: FUNCTIONING LIKE A PHYSICIST

In this session we'll look at exponentials and logarithms, how they relate to each other, and why they're relevant in physics.

EXPONENTIAL TRENDS

Lets say that some value, A, changed over time. We would write this rate of change as:

$$\frac{dA}{dt}$$

The variable t doesn't necessarily need to be time (but it often is)

Example: If A were **velocity,** its rate of change (the expression above) would simply be **acceleration -** and we know lots of equations to describe an object with constant acceleration!

But lets say A was something else. Instead of changing by a **constant amount** each second, it changed by a constant **factor.** Example: the factor could be **2** - in other words, it **doubles each second.** We would write this as:

$$\frac{dA}{dt} = 2A$$

So, for a **general case,** we can use **k** to represent the factor. We would say the rate of change of A depends on A itself; or it is **proportional to itself**:

$$\frac{dA}{dt} = kA$$

To express this in terms of A, we need to use **calculus.** The result, is the following:

$$A = A_0 e^{kt}$$

The method to find this is in the optional extras!

That equation is the **definition of exponential growth/decay.**

- **Growth** happens if the value of A **increases** over time **(k>0)**
- **Decay** happens if it **decreases** over time **(k<0)**

TASK 1: FIND THE ODD ONE OUT

5 MINS

One of the data sets in the following table (A, B, or C) is **not** changing exponentially with time. **Which one is it?**

t (s)	0	15	30	45	60	75	90
A	20	22	24	26	28	30	32
B	0.01	0.1	1	10	100	1,000	10,000
C	100	90	81	73	66	59	53

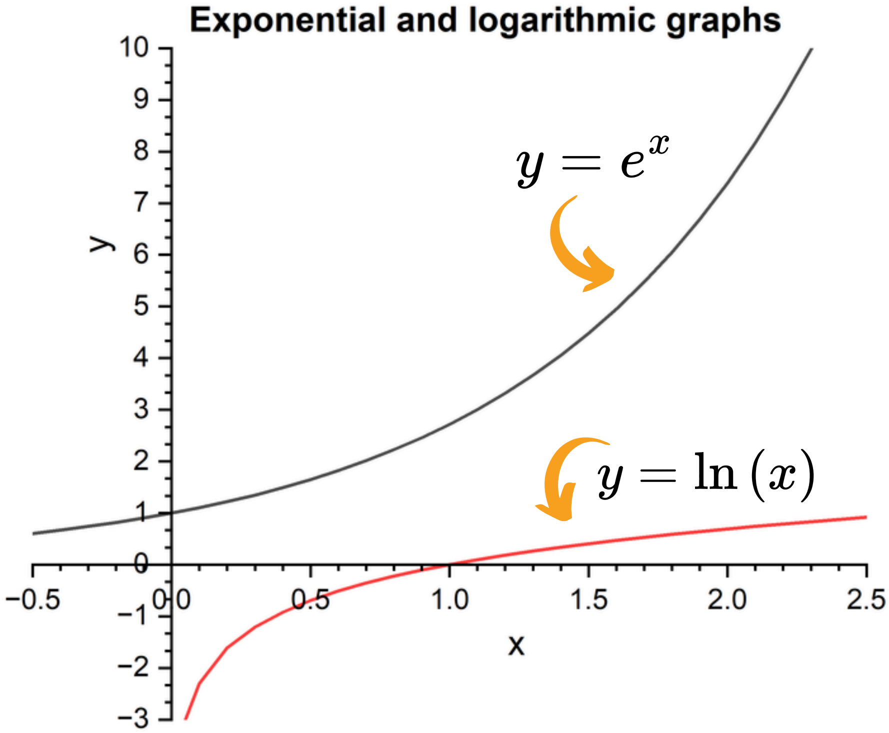
SESSION 5: FUNCTIONING LIKE A PHYSICIST

LOGARITHMS

Logarithms allows us to alter an exponential expression so it can be written in terms of the **power (kt).** This is often the value of interest. The equations below show how we would go **from an exponential expression to a logarithm.** Aloud, this would be said as ‘log base A of y is x’.

$$y = A^x \quad => \quad log_A(y) = x$$

Both functions are shown graphically below.



What’s the deal with e and ln?

The constant **e** comes about during the calculus step mentioned before, and it **comes up a lot** in sciences - especially **physics,** which relies heavily on maths. A **logarithm with base e** is called a **natural logarithm** and is expressed as **ln.**

Some useful log rules

$$log(a) + log(b) = log(ab)$$

$$log(a) - log(b) = log\left(\frac{a}{b}\right)$$

TASK 2: LOG PROBLEMS

10 MINS

Using these relation between logs and powers shown above on the log rules on right, **find x in the following equations:**

1. $log_{10}(x) + log_{10}(50) = 3$
2. $log_x(64) = 6$
3. $e^x = 20$

ADVANCED CONNECTIONS | CYSYLLTIADAU PELLACH

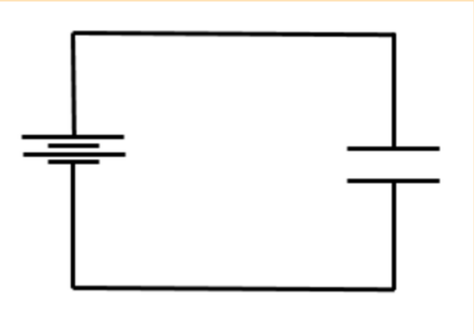
SESSION 5: FUNCTIONING LIKE A PHYSICIST

WHERE DOES THIS COME INTO PHYSICS?

This sort of mathematics comes up in science all the time! For A-level physics, there are two topics where logarithms/exponentials come up: **capacitance** and **radioactive decay**.

CAPACITANCE

A **capacitor** is a component of an electronic circuit consisting of **two metal plates** separated by an **insulator.** Each plate is connected to one end of a battery or cell, causing **electrons to flow** from one plate to another.



As one plate becomes more **positively** charged, it becomes **harder to remove the negative electrons** from it, just as it becomes harder to **add** them to the negatively charged plate. The **rate of flow of charge (current)** therefore depends on the **charge of the capacitor** - this is an **exponential relationship:**

$$Q = Q_0 e^{\frac{-t}{RC}}$$

RADIOACTIVE DECAY

The nucleus of an atom is made of **protons** (positive charges) and **neutrons** (neutral charges). We know from electrostatics that these should repel, but there is another force - the **strong nuclear force** - which holds nuclei together. This force can only act over short distances though, so **large nuclei** are much more **unstable.**

This stability can be improved by a process called **radioactive decay** in which nucleons can change and **particles are emitted** from the nucleus (called nuclear radiation). The decay is **probabilistic,** which means that for a single nucleus, we don't know exactly **when** it will decay - only the **probability** that it will over a given time. This means that, for an entire sample, the **number of atoms that decay per second (rate of decay)** is proportional to the **total number of remaining atoms** - this is an **exponential relationship:**

$$N = N_0 e^{-\lambda t}$$

TASK 3: REAL PROBLEMS

15 MINS

How would you express the two equations above in terms of **t**?

HINT: $\ln\left(e^{x}\right) = x$

SESSION 5: FUNCTIONING LIKE A PHYSICIST

OPTIONAL EXTRAS

HOW DO WE GET THE EQUATION ON THE FIRST PAGE?

$$\frac{dA}{dt} = kA$$

$$\int \frac{1}{A} dA = k \int dt$$

We can look this up in integration tables

$$\ln(A) = kt + C$$

$$A = e^{kt+C} = e^{C} \times e^{kt} = A_0 e^{kt}$$

We simply define this constant as the value of A when t=0

LOUDNESS - DECIBELS

We've all heard loudness measured in **decibels (dB)**, but did you know that decibels actually use a **logarithmic scale?** This means that 2 decibels louder is **twice as loud!**

$$\beta = 10log_{10}\left(\frac{I}{I_0}\right)$$

The reference intensity is the quietest sound a human can hear: $I_0 = 10^{-12} Wm^{-2}$

$\beta =$ loudness (dB)
$I =$ intensity of the sound wave
$I_0 =$ reference intensity

Normal conversation occurs at about **60 dB,** whereas a jet taking off 30 m away is **140 dB.** What is the **difference in intensity** of sound waves between these two sounds?